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ANALYSIS OF MEASURED AIRFOIL PRESSURE DISTRIBUTIONS

NASA CR-144901
VIZEX CR-75-6

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by

Raymond A. Piziali

October 1975

Prepared Under Contract No. NAS1-13274

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

In an earlier effort, a method for analyzing measured airfoil pressure distributions into their Glauert components was developed and applied. In general, the higher ordered components of the results obtained from the analyzed data were larger than could be rationalized. The present effort was undertaken to investigate and resolve the problem.

Using the techniques investigated in this effort, it was not possible to develop a computational procedure which would, in general, analyze airfoil pressure distributions into their Glauert components with reasonable accuracy. An alternate to the Glauert set of functions was considered but similar difficulties were encountered. Reasonable accuracy could not, in general, be obtained. The accuracy of the results was found to be influenced by many different parameters and aspects of the problem; however, there is one common denominator to them all and that is the non-orthogonality of the function-set chosen as the basis for the analysis.

The pressure distribution analysis problem is fundamentally that of analyzing a function into its constituitive components when given only a few isolated points on the function; the problem is compounded by the existance of errors contained in the data points. The difficulty of this task is closely tied to the function-set chosen as the basis for the analysis. Thus, the choice of the function-set, as well as the analysis technique, will influence the accuracy. Ultimately, however, there are two requirements on the basis function-set; interpretive usefulness and numerical accuracy.

The report discusses the difficulties encountered in attempting to analyze pressure distributions using two different basis function-sets.

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1.0 INTRODUCTION

Traditionally airfoil development has been carried out via two-dimensional steady-state wind tunnel tests and this has been quite satisfactory for fixed wing applications. Reasonable results have also been obtained by the application of such airfoils and airfoil data to helicopter rotors. However, when attempting to optimize airfoils for the helicopter application, the use of such results is not well justified because the remaining gains which may be possible will most likely be dependent on second order aspects of the airfoil aerodynamic operating environment. The operating environment for the helicopter airfoil is both non-steady and substantially different from the two-dimensional wind tunnel.

If the performance of an airfoil in the wind tunnel could be related to its performance on a rotor, then wind tunnel results could be more effectively used to develop improved helicopter airfoils. Furthermore, it would be helpful if more detailed descriptions of the actual aerodynamic operating environment of helicopter airfoils could be obtained. The need for such information was one of the objectives of a coordinated experimental research program initiated by the U. S. Army Aviation Material Laboratories (USAAVLABS) some years ago. As a part of this program, the chordwise rotor airfoil pressure distributions were measured in flight on several different helicopters (references 1, 2, 3 and 4) and in wind tunnel tests of steady and oscillating airfoils (references 5, 6 and 7). The results of this program have been useful and did confirm the non-uniformity of the rotor inflow, however the debates continued concerning subjects such as the actual aerodynamic angles

* The contract research effort which has lead to the results in this report was financially supported by USAAMRDL (Langley Directorate)

of attack experienced, the chordwise variation of the inflow (e.g., induced camber) and the applicability of wind tunnel data.

There was now a large body of data but inadequate means for extracting some of the desired additional information. Specifically there were no known rational quantitative means for analyzing airfoil pressure data. The data analyses were qualitative, i.e., "eyeball" comparisons of the flight and wind tunnel data (e.g., reference 8). What was now required was a rational method for quantitatively characterizing airfoil pressure distributions relative to their geometry and aerodynamic operating environment. This was basically the objective of an earlier effort by Tung and DuWaldt (reference 9) wherein a method was developed based on thin airfoil theory and then used to analyze measured pressure distributions into components corresponding to the terms of the Glauert series. In reporting the results of that effort the authors noted that the higher order coefficients evaluated from the pressure data were often much larger than expected. It was subsequently speculated that the methodology used may have been inadequate. Thus the effort, reported herein, was undertaken in an attempt to resolve this problem.

The specific objective of this effort was to correct the problems in the method for evaluating the Glauert coefficients from airfoil pressure distributions. The scope of this effort is limited to the linear operating range of the airfoils in steady-state or periodic operating conditions.

The ultimate objective of this effort was to develop a rational method for quantitatively characterizing airfoil pressure distributions relative to their geometry and aerodynamic operating environment. This capability would substantially enhance the value of pressure data in efforts to improve helicopter airfoils by enabling comparisons of pressure distributions to be made on a quantitative basis. Also the characteristics of the airfoil operating environment could now be determined from its measured pressure distribution.

SYMBOLS

A_m	-	coefficients of the Glauert function analysis
b	-	semi-chord
$B(\theta)$	-	pressure distribution due to angle of attack
ΔC_p	-	differential pressure coefficient
e	-	subscript indicating "effective" value
F_0, F_1	-	real part of unsteady pressure response function to r_0 and r_1
f_m	-	discrete or vector form of the basis function-set
G_0, G_1	-	imaginery part of unsteady pressure response function due to r_0 and r_1
k	-	reduced frequency $= b\omega/v$
M	-	number of function component considered by the analysis
NP	-	number of data points
P_m	-	$\Delta p(\theta,t)/2\rho V$
Δp	-	chordwise distribution of pressure
r_m	-	coefficients of the components of the stimulus
t	-	time variable
V	-	local free stream velocity
v	-	wake induced velocity normal to chord
w	-	sum of all velocities normal to chord
x_i	-	chordwise coordinate
α	-	angle of attack
$\dot{\alpha}$	-	pitch rate relative to mid-chord
γ_m	-	phase angle of stimulus, r_m
$\gamma(\xi,t)$	-	chordwise bound vorticity distribution
Γ	-	total bound circulation

θ	-	transformed chordwise coordinate;
ρ	-	density of the air
Φ_j	-	the components of the basis function-set
ω	-	frequency of airfoil oscillation

2.0 RESULTS AND CONCLUSIONS

This effort investigated the problem of analyzing measured airfoil pressure distributions into their Glauert components. While it can be demonstrated theoretically that airfoil pressure distributions are composed of these component functions, it was not possible to extract these components from the data with sufficient accuracy with the procedures investigated in this effort.

An alternate set of component functions was also considered as the basis for the analysis because they are more characteristic of the unsteady airfoil pressure distribution, $\Delta p(\theta)$, and thus required fewer components to represent the Δp. However, as with the Glauert functions, it was not possible to extract these components with reasonable accuracy by the procedures investigated herein.

The fundamental source of the poor accuracy and the difficulties encountered was traced to characteristics of the function-set considered as the basis for the analysis, that is, their non-orthogonality and their fundamental linear dependence. The non-orthogonality of the chosen basis set leads to a requirement that a large number of elements of the set be considered in the analysis to allow accurate determination of the component amplitudes. However, because the infinite basis set is linearly dependent, the sub-set considered for the analysis approaches linear dependence as the number of elements considered is increased. Fundamentally it was these two characteristics of the chosen function-sets which have prevented reasonable accuracy from being obtained in the analysis of airfoil pressure distributions.

The development of additional information relative to the general shape of a subject function which is given only at isolated points can be readily accomplished via numerous interpolation and curve fitting techniques. However, the analysis of the subject function into its constituitive components relative to a specific basis function-set is not as readily accomplished when given only isolated points on the function.

This effort was by no means exhaustive of possible function-sets which could be used in the analysis of airfoil pressure distribution data. Hence because of the potential benefits which would result from a technique for quantitatively analyzing airfoil pressure data, it is recommended that:

1) other basis function-sets be sought and evaluated which

   a) are orthogonal on the interval of the airfoil chord

   b) can be readily interpreted in terms of the airfoil geometry and operating environment

3) alternative analysis procedures be considered such as

   a) use of a minimization parameter other than the sum of the square errors

   b) imposing semi-empirical constraints on the analysis such as the integrals of the function, e.g., the lift and moment

   c) use statistical criteria to limit the number of components considered in the analysis.

3.0 TECHNICAL DISCUSSIONS

3.1 Statement of the Problem

How can quantitative information regarding the airfoil and its operating environment be extracted from the measured pressure distribution , $\Delta p(\theta)$, in a rational and consistent manner? For limited parameter ranges, the airfoil pressure distribution can be shown to be closely approximated by the sum of the component pressure distributions due to individual sources (or stimuli). Thus, if this set of individual pressure functions is used as a basis for analyzing the measured $\Delta p(\theta)$ then the mathematical problem is that of determining the coefficients of these components.

3.1.1 The Airfoil Pressure Distribution

The airfoil and its near wake can be considered as the "system" governing the airfoil "pressure-response" to a "stimulus". The stimulus and pressure-response in the airfoil problem are (respectively) analogus to the forcing function and dynamic response of a mass-elastic system.

The airfoil stimulus is, in general, time varying and is defined as the chordwise velocity distribution (normal to the chord) impressed on the airfoil from all sources except that induced by the airfoil near wake. It is generally dominated by the chordwise uniform normal velocity generated by the angle of attack, but it also includes the normal chordwise velocity distributions due the airfoil camber, airfoil motions, and freestream disturbances such as gusts, turbulence and interference velocities.

The general operating environment of an airfoil includes the freestream velocity, density, viscosity and the airfoil stimulus. Thus, in general, the pressure-response of the airfoil to a given stimulus will also be influenced by the Mach number and Reynolds number. However, within limited ranges of the parameters of the airfoil operating environment, the

pressure response to the individual components of the stimulus can beexpected to combine linearly. Thus a desirable characteristic of the set of functions used to analyze the pressure distribution, $\Delta p(\theta)$, is that the components of the set bear a one-to-one correspondence to the individual components of the stimulus. Such a set is suggested by thin airfoil theory (reference 9). This set, the Glauert functions, occupied the main thrust of this effort. A second, closely allied set, was also considered but to a lesser extent. These two pressure component sets, criteria for the choice of the set, and their relation to the airfoil pressure distribution are discussed in section 3.2.

3.1.2 The Mathematical Problem

For a specific choice of the function-set, the mathematical problem is that of analyzing a given Δp- function into these components of the set. Thus if

$$\Phi_j(x) \quad ; \; j = 1, 2, 3, \cdots$$

represents the chosen function-set and the given Δp-function was composed as

$$\Delta p(x) = \sum_{j=1}^{M} A_j \Phi_j(x) , \tag{1}$$

then the mathematical problem is to determine the M coefficients, A_j . However in practice the measured data, which experimentally define $\Delta p(x)$, are obtained only at discrete points, x_i , thus the mathematical problem is defined by the following set of equations

$$\Delta p(x_i) = \sum_{j=1}^{M} A_j \Phi_j(x_i) \quad ; \quad i = 1, 2, 3, \cdots NP \tag{2}$$

where $NP \geq M$ is the number of measured data points. Mathematically a unique solution exists for the M coefficients if the functions, $\Phi_j(x)$, are linearly independent, i.e.,

$$\sum_{j=1}^{M} C_j \Phi_j(x) = 0 \tag{3}$$

can be satisfied if, and only if, all the C_j are zero.

3.2 Pressure Function-Sets

There are several relevent considerations in making a choice of the function-set to be used for analyzing measured airfoil pressure distributions. The first is their interprative value;and the second is the total number of the member functions required to describe the pressure distribution. As a result of this investigation, a third and dominating characteristic of the function-set which must be considered is the orthogonolity of the set.

To be useful for interpretation, the analysis of the pressure distribution into components must provide information regarding the character of the flow response about the airfoil and the character of the airfoil operating environment. For example, relative to the flow response, can the pressure distribution analysis determine if the airfoil is operating in its linear range and, if not, which non-linear effects are present (i.e., stall, proximity to stall, what type of stall, separation bubble, shock wave, etc.)? Relative to the operating environment, what are the steady and dynamic "effective aerodynamic" angle of attack, pitch, rate, camber, and other higher ordered stimuli? Also what is the relative magnitude in the pressure distribution of the influence of the unsteady conditions?

The number of member functions required to describe the pressure distribution is of concern because the maximum number which can be calculated directly from the data is limited by the number of available data points on the chord. While additional "data" may be deduced from the original experimental data by curve fitting techniques, these "manufactured" data can lead to serious distortions in the determination of true components of the pressure distribution.

The use of a non-orthogonal set to analyze the pressure distribution leads to errors attributable to the fact that neglected terms affect those determined. The advantage of the use of an orthogonal set is the independence of each component of the set.

The dominating importance of this latter criteria in relation to the pressure analysis problem was not appreciated until well into the investigation. Hence the key criteria for the choice of function-sets was their interpretive value. The two function-sets chosen on this basis are presented in the following two sections. A third section discusses consideration associated with the choice of the leading terms of the function-sets. It is noted however, that the scope of this early effort is limited to the linear range only.

3.2.1 The Glauert Functions

Because the scope of this developmental effort is limited to the linear range of airfoil operation, and because linearized thin airfoil theory has been well demonstrated to yield good correlation with experiment, it is natural to look to the theory for suggestions as to the choice of a function-set. The solution procedure developed by Glauert suggests one possibility. The coefficients of this series are in one-to-one correspondence with coefficients of a cosine series expansion of the chordwise distribution of normal velocities (stimuli plus induced). Thus they would provide information about the airfoil operating environment and were therefore selected as the basis for the analysis. This selection was made in the initial effort reported in reference 9 and followed herein. This correspondence of the coefficients and their interpretation are discussed in the following.

The statement of the chordwise boundary condition of no flow through the airfoil is

$$4)\qquad w(x,t) - \frac{1}{2\pi}\int_{-b}^{+b} \frac{\gamma(\xi,t)\,d\xi}{x-\xi} = 0$$

where $w(x,t)$ is the chordwise distribution of the sum of velocities normal to the chord from all external sources, such as the airfoil motions, angle of attack, gusts, etc., including, as here written, the wake induced velocities in the time varying situation. Thus $w(x,t)$ represents the sum of the induced and stimulus velocity distributions. The integral in (4) represents the induced velocity distribution due to the airfoil bound vorticity, $\gamma(x,t)$.
If the distribution of bound vorticity is represented by the Glauert series

$$5)\qquad \gamma(\theta,t) = 2\left[A_0(t)\cot\frac{\theta}{2} + \sum_m A_m(t)\sin m\theta\right]$$

(where $x = -b \cos\theta$ is a convenient coordinate transformation) and substituted in (4), the result obtained is

6) $$w(\theta,t) = A_0 - \sum_m A_m \cos m\theta$$

Thus correspondence of the Glauert coefficients with those of a cosine expansion of the chordwise distribution of normal velocities is obtained by a term by term comparison of equations (5) and (6).

The relevence of the Glauert coefficients to the operating environment of the airfoil can be deduced from (6). It is observed that A_0 represents the magnitude of the uniform component of the normal velocities, w, over the chord and can be related to the instantaneous "effective angle-of-attack", α_e, thusly

7) $$\boxed{A_0 = V \sin \alpha_e}$$

where V is the local free stream velocity. Similarily A_1, represents the linear chordwise variation of these velocities and is related to the instantaneous "effective pitch rate", $\dot{\alpha}_e$, thusly

8) $$A_1 = b\dot{\alpha}_e$$

or related to the instantaneous "effective camber" (parabolic), thusly

9) $$y(x) = -\frac{A_1}{2V}(x^2 - b)$$

Likewise the successively high order coefficients represent higher ordered chordwise variations of the normal velocities, w. Therefore, if these coefficients can be obtained from the measured pressure distribution they will provide the type of information sought. Note that these instantaneous effective parameters would include both the wake induced and stimulus components.

When the Glauert functions are used to analyze the pressure distribution, the following is the representation of Δp,

10) $$\Delta p(\theta) = 2\rho V\left[\tilde{A}_0 \cot\frac{\theta}{2} + \sum_m \tilde{A}_m \sin m\theta\right]$$

From the theory, the chordwise distribution of pressure expressed in terms of the Glauert coefficients is (in the θ -coordinate)

$$11) \quad \Delta p(\theta,t) = 2\rho V\left[A_o(t)\cot\frac{\theta}{2} + \sum_{m=1}^{m} A_m(t)\sin m\theta\right] + 2\rho b\frac{\partial}{\partial t}\left[\left\{A_o(t) + \frac{1}{2}A_1(t)\right\}\theta + \left\{A_o(t) + \frac{1}{2}A_2(t)\right\}\sin\theta + \frac{1}{2}\sum_{m=2}^{m}\frac{1}{m}\left\{-A_{m-1}(t) + A_{m+1}(t)\right\}\sin m\theta\right]$$

It is thus observed, by comparison (10) and (11), that the analysis represented by (10) would result in coefficients, $\tilde{A}_m$, which would be the Glauert coefficients, A_m , only when the airfoil operating conditions are steady. However, if the unsteady motions of the airfoil are periodic, it is also possible to obtain the A_m from the $\tilde{A}_m$. The development of the equations for this transformation is outlined in Appendix I. Basically the assumption of periodic time variation allows the partial deviatives of (11) to be evaluated formally and a set of transformation equations to be derived.

In application of this function-set to the analysis of measured $\Delta p(\theta)$ the $\cot\theta/2$ function, which is singular at the leading edge ($\theta = 0$), is replaced by similar empirical function as discussed in the section "Leading Term of the Function Set".

3.2.2 The Unsteady Pressure Response Functions

Because the results of the $\Delta p(\theta)$ analysis using the Glauert functions were not satisfactory, another function-set was developed from thin airfoil theory which has two advantages over the Glauert functions. First, fewer terms would be required to represent the $\Delta p(\theta)$ and second, each function of this set is proportional to a corresponding stimulus component of the chordwise distribution of normal velocities rather than the combination of the stimulus and induced components as are the Glauert functions. The rational basis for this set is discussed below.

This function-set is derived from the fact that the total circulation, Γ , about an airfoil (in theory) depends only on the A_o and A_1 Glauert coefficients of the

airfoil bound vorticity distribution $\gamma(x,t)$ (equation 5), that is

$$\Gamma = \int_{-b}^{+b} \gamma(x,t)\,dx = 2\pi b\left[A_0(t) + \tfrac{1}{2}A_1(t)\right] \tag{12}$$

In discussion of the above section, it was shown that $w(x,t)$ (the chordwise distribution of velocities normal to the chord) and the coefficients $A_m(t)$ of its cosine representation are, in general, the sum of an induced and a stimulus component, say $A_m = v_m + r_m$. Thus equation (6) becomes

$$w(\theta,t) = \left(v_0 - \sum_m v_m \cos m\theta\right) + \left(r_0 - \sum_m r_m \cos m\theta\right) \tag{13}$$

Now consider the case where there is only an r_0 stimulus (say $r_0 = \bar{r}_0 \cos \omega t$) then it is clear that all the other A_m will contain only an induced part (i.e. $A_m = v_m$). This will be due to the airfoil wake generated by the time variation of r_0 . When this resulting set of A_m is then substituted in (11), the corresponding pressure distribution, P_0 , due to an r_0 – only stimulus is obtained. This P_0 is the first function of the set. The second function of the set, P_1 , due to an r_1 – only stimulus is obtained similarily. Now because the wake vorticity is determined by the time-rate-of-change of the Γ , and Γ depends only on A_0 and A_1 the higher ordered r_m (i.e. $m > 1$) will not generate any wake nor any A_m at orders other then m . That is an r_m – stimulus will yield only an A_m ($= r_m$) and when substituted in (11) yield a corresponding P_m . Because these pressure functions each contain all the unsteady aerodynamic effects as per (11) fewer terms than the Glauert series are required to represent Δp i.e., they are characteristic of the airfoil pressure response. They are referred to herein as the unsteady pressure response functions (UPRF) and when the time variation is periodic they are functions of the reduced frequency, k . They reduce to the Glauert functions in the steady-state case when $k = 0$.

The r_0 and r_1, correspond respectively to the effective angle of attack stimulus, α_e, and the effective pitch-rate (relative to mid-chord) stimulus, $\dot{\alpha}_e$. The corresponding UPRF are obtained from integral representations in the literature (e.g. reference 10). For the r_0 & r_1 stimuli

14) $$P_0(k,x,t) = F_0(k,x)\cos\omega t - G_0(k,x)\sin\omega t$$

where $$F_0(k,x) = \sqrt{\frac{b-x}{b+x}}\left[Re\{C(k)\}\right]$$

and $$G(k,x) = \sqrt{\frac{b-x}{b+x}}\left[k(b+x) + Im\{C(k)\}\right]$$

15) $$P_1(k,x,t) = F_1(k,x)\cos\omega t - G_1(k,x)\sin\omega t$$

where $$F_1(k,x) = \sqrt{\frac{b-x}{b+x}}\left[1 - Re\{C(k)\} + 2(b+x)\right]$$

$$G_1(k,x) = \sqrt{\frac{b-x}{b+x}}\left[kx(b+x) - Im\{C(k)\}\right]$$

$k \sim$ reduced frequency

$C(k) \sim$ Theodorsen function

$\omega \sim$ frequency

$\sqrt{\frac{b-x}{b+x}}$ Correspondes to $\cot \theta/2$

Examples of these two UPRF are presented in figure 1 for $k=0.211$. The procedure for using this function-set is similar to that for the Glauert function. The procedure and equations are presented in Appendix II.

3.2.3 Leading Term of the Function Set

The leading term for the pressure distribution representations obtained from thin airfoil theory is singular at the airfoil leading edge because the airfoil is represented with zero radius at the leading edge. With the exception of the near vicinity of the leading edge, the solution of thin airfoil theory have been demonstrated to be quite good for the linear operating range of the airfoil. This term represents the pressure distribution due to a steady r_0 stimulus (i.e.,

angle of attack). In the chordwise $x-$ coordinate, it is

$$\sqrt{\frac{b-x}{b+x}}$$

while in the $\theta-$ coordinate (where $x = -b\cos\theta$) it is

$$\cot \theta/2$$

Using this singular function in the analysis of pressure distributions would result in significant errors if there were data points near the leading edge.

One possible alternative is to apply a thick airfoil correction to $\cot \theta/2$. Roshko (reference 10), for example, develop a correction of this type utilizing an ellipse (fit to the forward part of the subject airfoil) plus a flat plate fin extension (to obtain the desired chord length).

Another possibility, for this leading term, is to use the measured steady-state pressure distribution obtained for the subject airfoil at a small angle-of-attack and normalized by this angle. The typical correlation of $\cot \theta/2$ with measured steady-state small-angle Δp is observed in figure 2. For this developmental effort measured Δp were used to define the leading term and is designated as $B(\theta)$.

It is noted that a characteristic of this leading term $B(\theta)$, in the $\theta-$ coordinate system is that it will have a zero slope at $\theta = 0$. This is simply the result of the coordinate transformation $x = -b\cos\theta$ and the fact that, in general, the airfoil surface pressure distribution is a continuous function of the "surface coordinate" around the leading edge.

3.3 Solution Procedure

The analysis is defined by the set of equations,(2). If the number, M, of functional components to be considered by the analysis (i.e., to be evaluated) is equal to the number, NP, of data points defining the measured Δp function, then equations (2) are a set of NP linear simultaneous equations in the NP unknown coefficients. However, when the number, $\overline{M}$, of components to be evaluated from the measured Δp is less than the number of data points, equations (2) will represent an overdetermined system; that is, there are more data (and thus equations) than unknown coefficients. This "extra" data can effectively be used to determine the, M, unknown coefficients by application of the method of least squares. The function, $\overline{\Delta p}$, defined by these, M, components will be the best "fit" to the data, $\Delta p(\theta_i)$, in the sense that the average square error (residual) at the data points

$$16) \qquad \eta^2 = \frac{1}{NP}\sum_{i=1}^{NP}\left[\overline{\Delta p(\theta_i)} - \Delta p(\theta_i)\right]^2,$$

will be minimumized and the "fit" will not pass through the data points. However, when $M = NP$, the residual will be zero and the "fit" will pass through the data. That is when $M = NP$ the results will be the same as those obtained directly from equations (2).

By applying the principle of least squares, the equations (2) to be solved for the unknown, A_m, become

$$17) \qquad \sum_{m=1}^{M} A_m \left\{\sum_{i=1}^{NP}\Phi_m(x_i)\Phi_k(x_i)\right\} = \sum_{i=1}^{NP}\Phi_k(x_i)\,\Delta p(x_i)\,; \quad k=1,2,\cdots M$$

where the number, NP, of data points must be greater than or equal to the number, M, of functions considered. Equations (17) are a set of, M, linear simultaneous equations in the, M, unknown, A_m, where the coefficients,

$$\Phi_{mk} \equiv \left\{\sum_{i=1}^{NP}\Phi_m(x_i)\Phi_k(x_i)\right\},$$

of the equations are determined only by the function-set used by the analysis and the data point locations, x_i. Thus, for application to a given set of measured Δp (all obtained

at the same x_i) the coefficient-matrix $[\Phi_{mk}]$ of (17) need only be evaluated and inverted once. The analysis of each measured $\Delta\rho$ is then obtained by simply evaluating the right-hand side of (17) and pre-multiplying it by the inverse.

4.0 DISCUSSION OF DIFFICULTIES ENCOUNTERED

Unless otherwise stated, and only for the purposes of the following discussions, the measured pressure data is considered to be free from experimental error and to coincide with the actual airfoil pressure distribution (the subject function) at their respective chordwise locations.
The following is a discussion of the various aspects of the overall problem, the difficulties encountered, their interelations and their relation to the various aspects of the problem as posed. This investigation was begun recognizing that there was a difficulty with the analysis as developed and reported in reference 9. However, the essential nature of the difficulty was not fully appreciated. Thus, many false starts were made and blind alleys traveled in search of an adequate analysis procedure. The following discussion is neither exhaustive nor conclusive but simply a presentation of the present hindsight views of the problem.

4.1 Discussion

The problem objective is to analyze airfoil pressure distributions, Δp, into their Glauert function components. That is, determine the coefficients of a series expansion of the airfoil pressure function where the basis functions for the representation are the Glauert functions. The pressure distribution ("subject function") is a continuous function and an infinite number of points are required to define it. However, this continuous subject function is not available for the analysis. The data are available only at a relatively few isolated points and thus define a "discrete function" (or vector) with only a finite number of components. It is clear that an unlimited number of continuous functions (in addition to the subject function) can be made coincident with these data points. Thus the subject function

is not uniquely defined and there is the following question: Which of the many possible continuous functions has been (or is being) analyzed? The task, therefore, is not to determine the coefficients of any smooth "fit" to the data but specifically the coefficients for the subject function from which the data are obtained.

The finite number of available components for this data-defined discrete function limit the analysis to this number of components. That is, the coefficients for only NP components can be evaluated (where NP is the number of data points). If the function-set used as the basis for the analysis is orthogonal then it is mathematically possible to determine the first NP coefficients because each is independent of the others. However, if the function-set is non-orthogonal, then all the components of the subject function are inter-dependent and thus all present must be evaluated simultaneously to obtain the correct values. That is, of all components of the subject function are not included in the analysis then the neglected components will give rise to errors in the coefficients of those included. These errors are herein referred to as truncation errors. In general, the number of components (in terms of the chosen function-set) actually contained in the pressure function will be greater than the number of data points. Thus, some components must, of necessity, be neglected. Non-orthogonality of the basis for the analysis will then give rise to truncation errors in the computed components due to the neglected components.

The number of components of the chosen function-set actually contained in the subject pressure function (i.e., required to represent it) will, of course, depend on the function-set chosen. If the chosen set is characteristic of the airfoil pressure function, Δp , then relatively few components will, in general, be required to represent it. The Glauert functions are characteristic of the <u>theoretical</u> airfoil $\Delta p(\theta)$. This, primarily because of the leading angle-of-attack pressure function. If, for example, the set of sine functions were the chosen

set, then theoretically an infinite set of these components would be required just to represent the angle-of-attack function component of $\Delta\rho(\theta)$. Thus the specific function used for this angle-of-attack function will be crucial with regard to the total number of components required to represent $\Delta\rho(\theta)$. That is, that portion of the actual $\Delta\rho(\theta)$ which cannot be represented by the chosen angle-of-attack function must be represented by the remainder of the function set.

Because of the practical limitation of a finite number of data points, the analysis task must work with finite-component discrete functions rather than the continuous subject function and the continuous functions of the chosen function-set (basis). These finite-component discrete functions are (or can be considered as) finite NP- dimensional vectors where the magnitude of each component of the vector is the magnitude of the corresponding function at each chord point and the vector space is the specific set of chord points. That is the discrete function is a plot of the vector components. Given a set of chord points, the pressure function defines the "measured vector" and the basis function-set defines a "basis vector-set". However, it is noted that orthogonality of the function-set will not assure orthogonality of the corresponding vector-set. The skewness (degree of non-orthogonality) of the resulting vector-set will be determined not only by the functions but also by the specific chord points used to define the vectors. That is, for a given function-set the skewness of the corresponding vector-set will vary with the number and distribution of the chord points used. Thus, the interdependence of the coefficients and the truncation errors due to neglected components (discussed above) will, for the actual analysis, be determined by the skewness of the vector-set (rather than the function set).

Skewness of the basis vector-set will influence the "condition" of the governing equations of the analysis. The governing equations (section 3.3) are a system of linear equations. When small perturbations in the elements of the coefficient matrix of these equations or in the right-hand sides or in the process of solution of this system have little influence on the solution, the system is termed "well-conditioned"; however, if the influence is large, i.e., the solution is sensitive to these perturbations, then

the system is "ill-conditioned". As stated in reference 12, "all methods of solving an ill-conditioned system of linear equations are generally bad". Generally as the skewness of the basis is increased, the condition of the system deteriorates. For linear systems, ill-conditioning occurs when the matrix of coefficients is nearly singular, i.e., when some rows or columns are almost linearly dependent. When the system is ill-conditioned it becomes numerically very difficult to obtain an accurate inverse. However, even if an exact inverse is available, the solution will be very sensitive to perturbutions of the right-hand side of the equations which are determined by the data. Thus small changes in the data will result in significant changes in the results.

The Glauert function-set (section 3.2.1) is composed of a leading term ($B(\theta)$), representing the $\Delta p(\theta)$ due to angle-of-attack plus the sine functions $\sin n\theta;\ n=1,2,\cdots NS$. This set of functions is non-orthogonal because of the leading function. Also this set of functions has the characteristic that as NS increases, the set approaches linear dependance. (The UPRF-set (section 3.2.2) also have this characteristic). Because the set of sine functions (for $n \rightarrow \infty$) are a complete set on the interval $0 \leq \theta \leq \pi$, all bounded integrable functions on this interval may be represented by a linear combination of them. If NS is large enough, the $B(\theta)$ of the Glauert function-set can be represented by a linear combination of the sines and thus the set will be linearly dependent. In practice, however, the problem is not that the set of functions will be linearly dependant but that the governing equations become increasingly ill-conditioned as NS is increased and the set approaches linear dependance. This leads to the numerical and sensitivity problems of ill-conditioned systems discussed above.

The limitation (due to the finite number of data) on the number of components considered by the analysis, and thus the resulting truncation errors, could possibly be circumvented if additional (psuedo) data were obtained from the measured data by interpolation. This however, can lead to two difficulties. First, as described above for the Glauert and UPRF-sets, if the number of functions used is increased, the governing equations become ill-conditioned. Second,

the interpolated data will not necessarily coincide with the subject function, i.e. false data can be introduced. Depending on the magnitude of the interpolation error in this false data and the sensitivity of the governing equations (which also will be increased by increasing the number of components.), large errors in the computed coefficients can occur.

Another possibility for circumventing the difficulties related to the non-orthogonality of the function-set is to orthogonalize the set. The analysis of the subject function could then be performed relative to the orthogonal set and would be free from these difficulties. The coefficients of the expansion relative to the orthogonal set could then be transformed back to the non-orthogonal set. There are, however, two difficulties which are presented by this approach. In general, many more functional components will be required to represent the airfoil pressure function in the orthogonal set than in the non-orthogonal set. All of these additional components (in the orthogonal set) must be included in the analysis if the truncation errors are to be avoided because, in general, each component in the non-orthogonal set will depend on all components in the orthogonal set. But, as described above, the available information (i.e., the number of data points) defining the subject function is very limited and thus it is not possible to expand the number of components to be considered without encountering other problems. Thus, this approach is not a viable one for avoiding the truncation error difficulty. Furthermore it is not useful for avoiding the problem of ill-conditioning. It can be shown that if the governing equations for the analysis relative to the skewed set are ill-conditioned then the equations for transforming the results back to the skewed set from the orthogonal set will also be ill-conditioned.

The difficulties discussed above are encountered in the absence of error in the data. If the data contain error, the difficulties are severly compounded because the relatively few data are the only information available concerning the unknown subject function. Recall, even if exact they do not adequately define the subject function. When

the data contain error they, in effect, are obtained from a function which differs from the subject function by an unknown amount. Thus, in general, while it is theoretically possible to calculate a set of coefficients from a given set of data using a given set of linearily independent functions, in practice the resulting coefficients will bear little resemblence to the true coefficients of the subject function from which the data were obtained except under special conditions.

The coefficients of the representation of the subject function in terms of the chosen function-set can be obtained when the number of functions of the set included in the analysis encompasses all that are present in the subject function or if there are neglected components, they are orthogonal to all the components which have included in the analysis. These conditions will always be satisfied if the chosen set is orthogonal.

The above statements are true when the data points actually coincide with the subject function (i.e., no errors). If there are errors in the data, then the influence on the resulting coefficients will depend on their distribution. If they are distributed such that they represent a perturbation from the subject function which is orthogonal to each of the component functions which have been considered in the analyses, then they will not influence the results. If, however, this error perturbation on the subject function is not orthogonal to the included components then, of course, the error will contribute to them.

In considering all the above ramifications of the problem, it becomes clear that the non-orthogonality of the basis functions-set can influence the analysis in many ways. Thus, it is desirable that the chosen function-set not only be useful interpretation of the airfoil pressure distributions (as discussed in section 3.2) but also be orthogonal on set of chord points at which the data are given.

4.2 Examples

The following examples are presented to illustrate the nature and severity of some of the difficulties discussed in the above section. For this purpose, a synthesized pressure distribution, $\Delta C_p(\theta)$, approximating the NACA 4415 at $\alpha = 0.1$ radian is synthesized from the first nine Glauert functions so the true content of the subject function is known. The values of these coefficients are presented in Table 1 and the function is presented as the curve in figures 3 and 4.

Thirteen "data points" are selected from this subject function at the same chordwise locations used for the wind tunnel tests of reference 6. If these data are analyzed considering at least the first nine components of the Glauert function-set, then the true values of the coefficients are obtained because <u>all</u> the functional components actually in the data are included in the analysis (none are neglected).

If these thirteen data points (the $+$ symbols in figure 3) are interpolated to obtain eighteen equally spaced points, they will not be on the subject function as shown in example 1 of figure 3. (A spline-fitting interpolation procedure was used to obtain these points.) These interpolated (psuedo) data are from a different function whose composition includes more than the first nine Glauert components. Thus when this psuedo-data is analyzed considering only the first nine components of the Glauert function-set, those components beyond the first nine will have been neglected. Because the function-set is non-orthogonal, "truncation errors" result. The magnitude of these errors is evident in the resulting coefficients presented in Table 1 where they are compared to the true values of the subject function.

In the above example, the deviation of the psuedo-data from the subject function are relatively small except at one point. Yet the errors in the results are large, thus the degree of sensitivity of the solution to the data is observed to be large. This sensitivity is demonstrated in example 2 where eighteen equally spaced data points were

taken from the subject function and a -5% error was introduced in one point as shown in figure 4. When a ten component analysis is made of this "data" the resulting coefficients, presented in Table 1, contain significant errors. Whether the data is considered to actually contain components neglected by the analysis or errors in the individual data points, the results are the same. In practice both neglected components and data error will be present simultaneously and must be coped with.

The truncation errors result from the non-orthogonality (or skewness) of the basis used for the analysis. As discussed in section 4.1, the basis functions are not used directly but rather a set of discrete functions (vectors) defined by them and the chordwise distribution of data points is the actual basis for the analysis. The direction cosines (dot product of unit vector pair) of a vector pair is a measure of their skewness (or the generalized angle between them). The value of the direction cosine range between zero and one with a value of one indicating the two vectors are parallel (i.e., the one discrete function of the pair is a constant times the other) and a value of zero indicating that the two are orthogonal.

The influence of the chordwise distribution of the data on the skewness of the vector set defined by the Glauert functions is illustrated in Table 2 for two distributions. Presented are the direction cosines (relative to the remainder of the set) of the first two vectors of the set, i.e., the one corresponding to the angle-of-attack function, $f_0 \equiv B(\theta)$, and the one corresponding to the $f_1 \equiv \text{Sine } 1 \cdot \theta$. The first case is for eighteen equally spaced points along the chord. Because the sequence of sine functions $(f_n \equiv \text{Sine } n \cdot \theta,\ n = 1, 2, 3, \cdots)$ are orothogonal and the points are equally spaced. The resulting vectors are orthogonal as is evident by the small values (1×10^{-6}) of the corresponding direction cosines (i.e., for $f_1 \cdot f_n$). The skewness of $f_0 \equiv B(\theta)$ relative to the remainder of the set is evident by the relatively large values of the $f_0 \cdot f_n$. For the second case, the thirteen non-equally spaced points of reference 6 are used.(shown in figure 3) Here in column 2 the non-zero values ($f_1 \cdot f_n$) indicate that the vectors corresponding to the sine functions are no longer orthogonal, and

the values of the direction cosines ($f_0 \cdot f_n$) in column 1 are generally larger than for case 1 indicating that the skewness of $B(\theta)$ relative to the sine functions is greater in the second case.

The truncation errors resulting from non-orthogonality are also dependent on the components of the subject function neglected by the analysis. A relatively large number of Glauert components are required to represent the $\Delta C_p(\theta)$ because of the shed wake effects, thus, in general, neglected components are ever present in the subject function relative to this function-set. As discussed in section 3.2.2, the UPRF set are theoretically characteristic of the $\Delta C_p(\theta)$ and thus, in general, only a few components should be required and the neglected components, if any, should be small and few. Thus the use of this set was explored using a synthesized $\Delta C_p(\theta)$ corresponding to an experimental case of reference 6 for an airfoil occilating in plunge only (i.e., the only stimulus present is $r_0 = 21.75$ with $\gamma_0 = -90°$).

The chordwise distribution of the magnitude of this $\Delta C_p(\theta)$ and its phase distribution are presented in figure 5 as the dashed curves. The corresponding experimental data (11 points) are also shown for comparison. As explained in section 3.2.2, the $\Delta C_p(\theta)$ is represented in terms of the unsteady pressure response functions (UPRF) due to the stimuli, r_m . Thus as shown in Appendix II, the stimulus amplitudes, r_m , become the coefficients of the representation sought by the analysis and for the first two (i.e., r_0 & r_1) there are redundant solutions, one each, from the sine and the cosine components of the $\Delta C_p(\theta)$ (equations 3a and 3b of Appendix II). The results of analyzing this synthesized $\Delta C_p(\theta)$ using "data" selected from it at the eleven chordwise locations of reference 6 are presented in Table 3. (For r_0 & r_1 the redundant solutions are presented.) Three analyses were made progressively increasing the number, m , of stimulus components considered by the analysis. It is observed that for $m = 1$ the results are exact but that they deteriorate as more components are considered.

A second $\Delta C_p(\theta)$ was synthesized for a combination of r_0 & r_1 stimulus, i.e., a combination of plunge and pitch. The results of three analysis considering progressively more components are presented in Table 4. Here the results are observed to be poor for all three cases. The experimental data shown in figure 5 corresponding to the first $\Delta C_p(\theta)$ were also analyzed in a similar manner and the results are presented in Table 5. Here the results are, again, very poor and the differences between the results in Tables 3 and 5 correspond to the differences between the synthesized and experimental $\Delta C_p(\theta)$ presented in figure 5. These differences in these results are in part due to the influence of data errors and/or truncation errors for the UPRF set. However the direction cosines of the UPRF set, presented in Table 6, for the real and imaginary parts corresponding to r_0 & r_1, stimuli reveal that they are an extremely skewed set and therefore the solutions are extremely sensitive to minor perturbations in the data and/or the numerical processing. Thus while the UPRF set is characteristic of the unsteady airfoil pressure response, i.e.,very few terms of the UPRF set are required to represent the $\Delta C_p(\theta)$, the functions are poorly suited for analysis.

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TABLE 1

Comparisons of Actual and Calculated Values of Glauert Coefficients, A_m, in a Synthesized $\Delta C_p(\theta)$

Two Examples:

No. 1 Interpolation of Data to Equal Spacing

No. 2 5% Error on One Data Point

m	ACTUAL VALUE A_m	EXAMPLE No. 1 (fig. 3)	EXAMPLE No. 2 (fig. 4)
0	25.00	-10.95	19.38
1	40.75	101.34	50.04
2	6.925	75.28	17.19
3	1.3325	50.20	8.36
4	-1.045	37.82	4.44
5	-1.025	26.44	2.60
6	0.23375	19.38	2.60
7	0.635	12.67	2.09
8	0.007475	8.80	1.00
9	0.0	5.86	0.753
10	0.0	4.68	0.533

TABLE 2

Direction Cosines, $f_j \cdot f_m$, of First Two Glauert Functions Relative to Remainder of Set for Two Cases

	Case No. 1 18 Equally Spaced Points		Case No. 2 13 Non-equally Spaced Points of Ref. 6	
m	$f_0 \cdot f_m$	$f_1 \cdot f_m$	$f_0 \cdot f_m$	$f_1 \cdot f_m$
0	1.0	$\approx 1\mathrm{x}10^{-6}$	1.0	.596
1	.280	1.0	.596	1.0
2	.316	$\approx 1\mathrm{x}10^{-6}$	.673	.243
3	.227		.492	.0975
4	.186		.223	-.170
5	.132		.0747	-.225
6	.093		.0703	.0763
7	.062		.0767	.0302
8	.043		.0855	.0200
9	.027		.0875	.0665
10	.011		.0670	.0743

TABLE 3

Airfoil Stimulus Amplitude and Phase (r_m, γ_m) Calculated from Synthesized $\Delta C_p(\theta)$ Due Only to Plunge Oscillation.

(r_0, γ_0) = (21.75, - 90°); k = 0.211

M	Eqn.	(r_0, γ_0)	(r_1, γ_1)	(r_2, γ_2)
1	Cos.	(21.75,-90°)	—	—
	Sin.	(21.75,-90°)	—	
2	Cos.	(16.1,-96°)	(0.6,0°)	—
	Sin.	(27.0,-33°)	(2.4,-90°)	
3	Cos.	(15.7,-97°)	(0.6,-10°)	(0.2,-90°)
	Sin.	(26.7,-34°)	(2.3,-90°)	

TABLE 4

Airfoil Stimulus Amplitude and Phase (r_m, γ_m) Calculated from Synthesized ΔC_p Due to Plunge and Pitch Oscillations

(r_0, γ_0) = (21.75,-90°); (r_1, γ_1) = (4.35,0°); k = 0.211

M	Eqn.	(r_0, γ_0)	(r_1, γ_1)	(r_2, γ_2)
1	Cos.	(85.2,-78°)	—	—
	Sin.	(25.8,-36°)	—	
2	Cos.	(16.3,-100°)	(7.2,-7°)	—
	Sin.	(26.8,-38)	(5.3,-1°)	
3	Cos.	(12.7,-102°)	(5.3,0°)	(4.4,-45°)
	Sin.	(23.9,-44°)	(4.7,-23°)	

TABLE 5

Airfoil Stimulus Amplitude and Phase (r_m, γ_m) Calculated from a Measured ΔC_p Due to Only Plunge Oscillation

(r_o, γ_o) = (21.75-90°); k = 0.211

m	Eqn.	(r_o, γ_o)	(r_1, γ_1)	(r_2, γ_2)
1	Cos.	(44.4,-86°)	—	—
	Sin.	(43.3,-19°)	—	
2	Cos.	(70.5,-108°)	(37.2, 72°)	—
	Sin.	(539.2, 16°)	(101.1,-150°)	
3	Cos.	(475.7,-62°)	(474.6, 90°)	(202.8,-113°)
	Sin.	(2264., 24°)	(1365.,-175°)	

TABLE 6

Direction Cosines of the First Two Components of the UPRF set, i.e., Corresponding to r_o & r_1 Stimulus

	F_o	G_o	F_1	G_1
F_o	1.0000	-.95919	.79785	.99117
G_o	-.95919	1.0000	-.59484	-.94489
F_1	.79785	-.59484	1.0000	.80325
G_1	.99117	-.94489	.80325	1.0000

F_m are the real part of the P_m due to r_m

G_m are the imaginary part of the P_m due to r_m

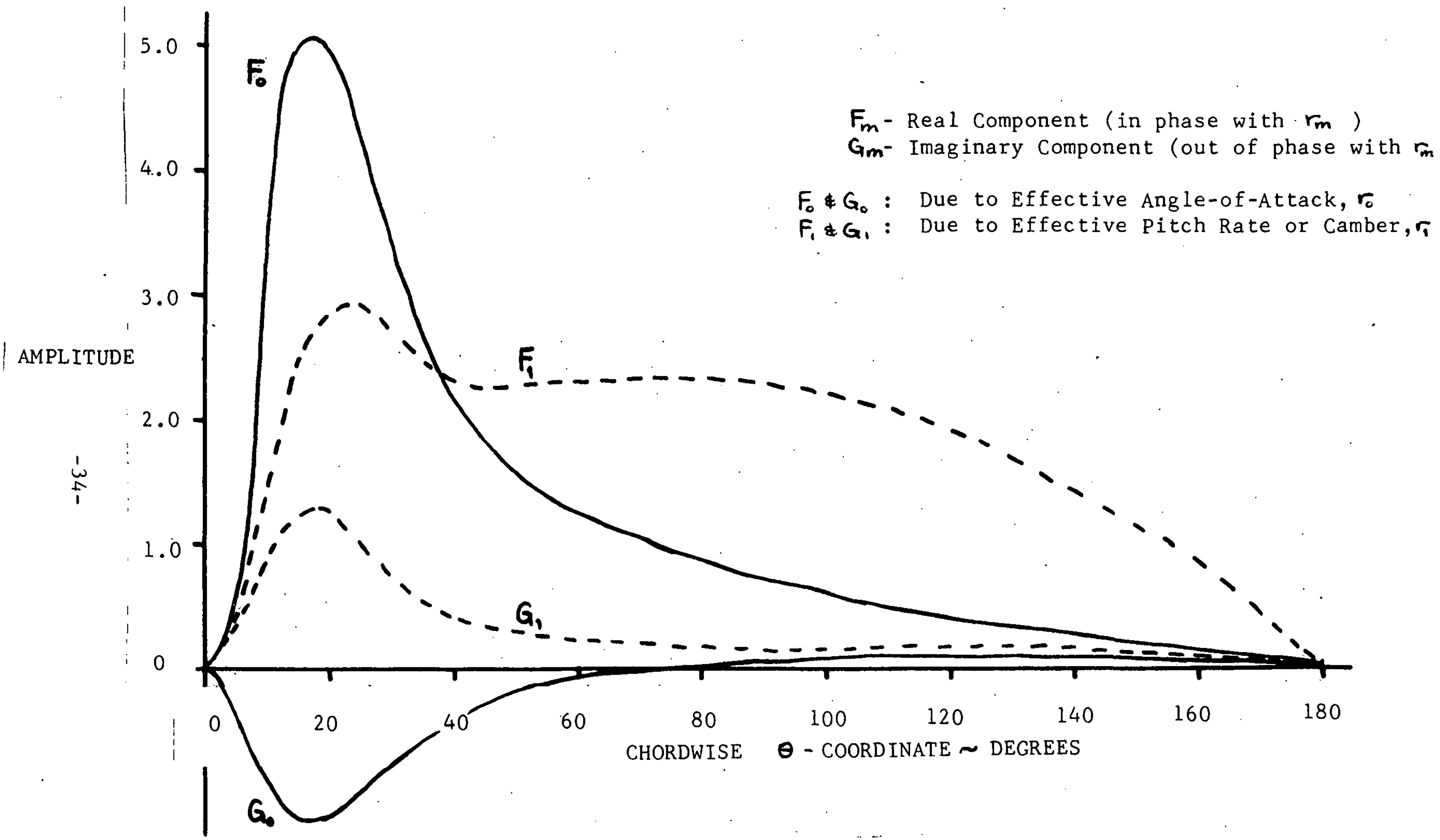

FIGURE 1. Unsteady Pressure Response Functions @ Reduced Frequency, k = 0.211

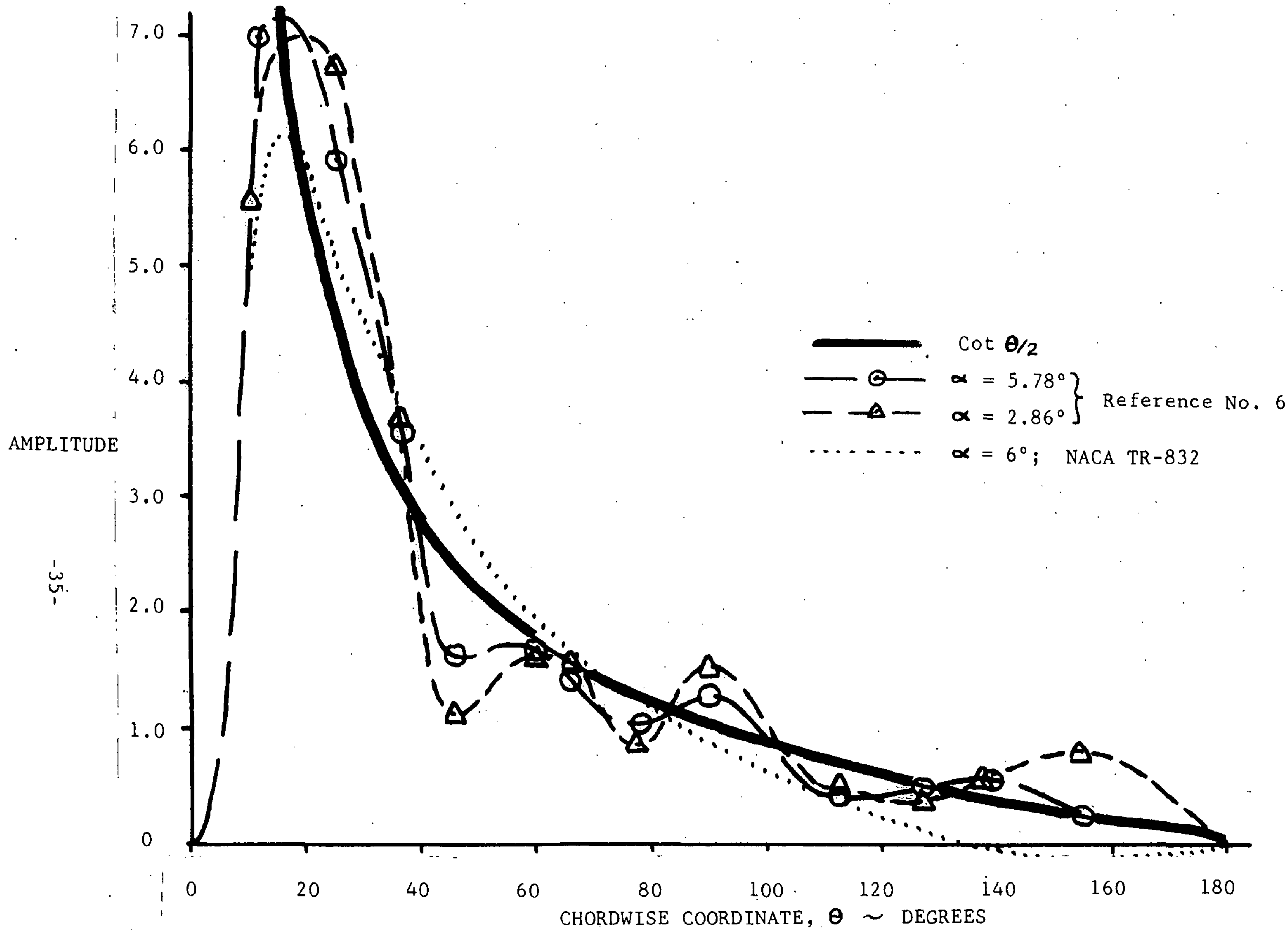

FIGURE 2. Comparison of Measured NACA 0012 Airfoil Pressure Distributions with Cot $\theta/2$

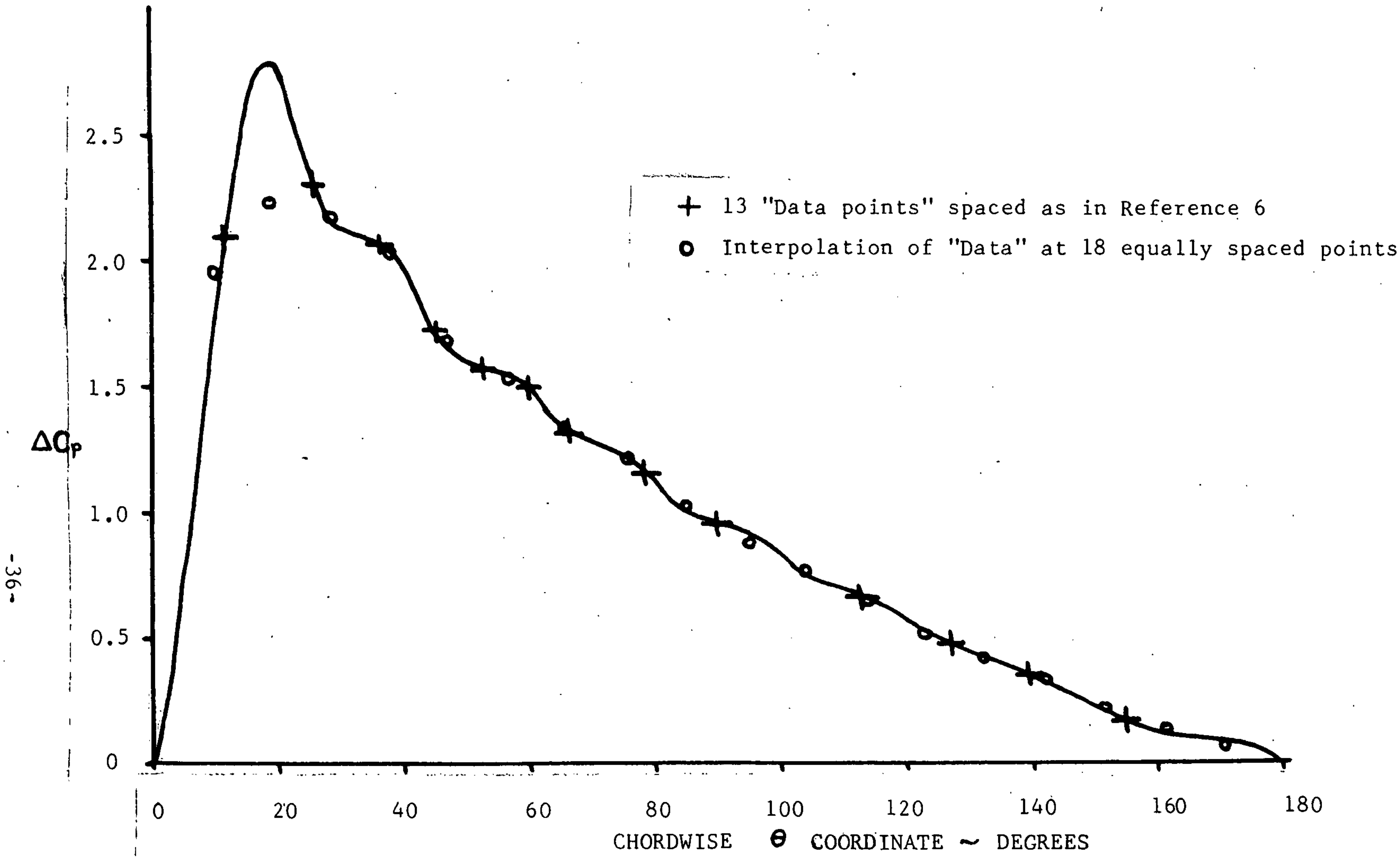

FIGURE 3. Example 1: Interpolation of "Data" to Equal Spacing Synthesized $\Delta C_p(\theta)$ of NACA 4415 at $\alpha = 6°$

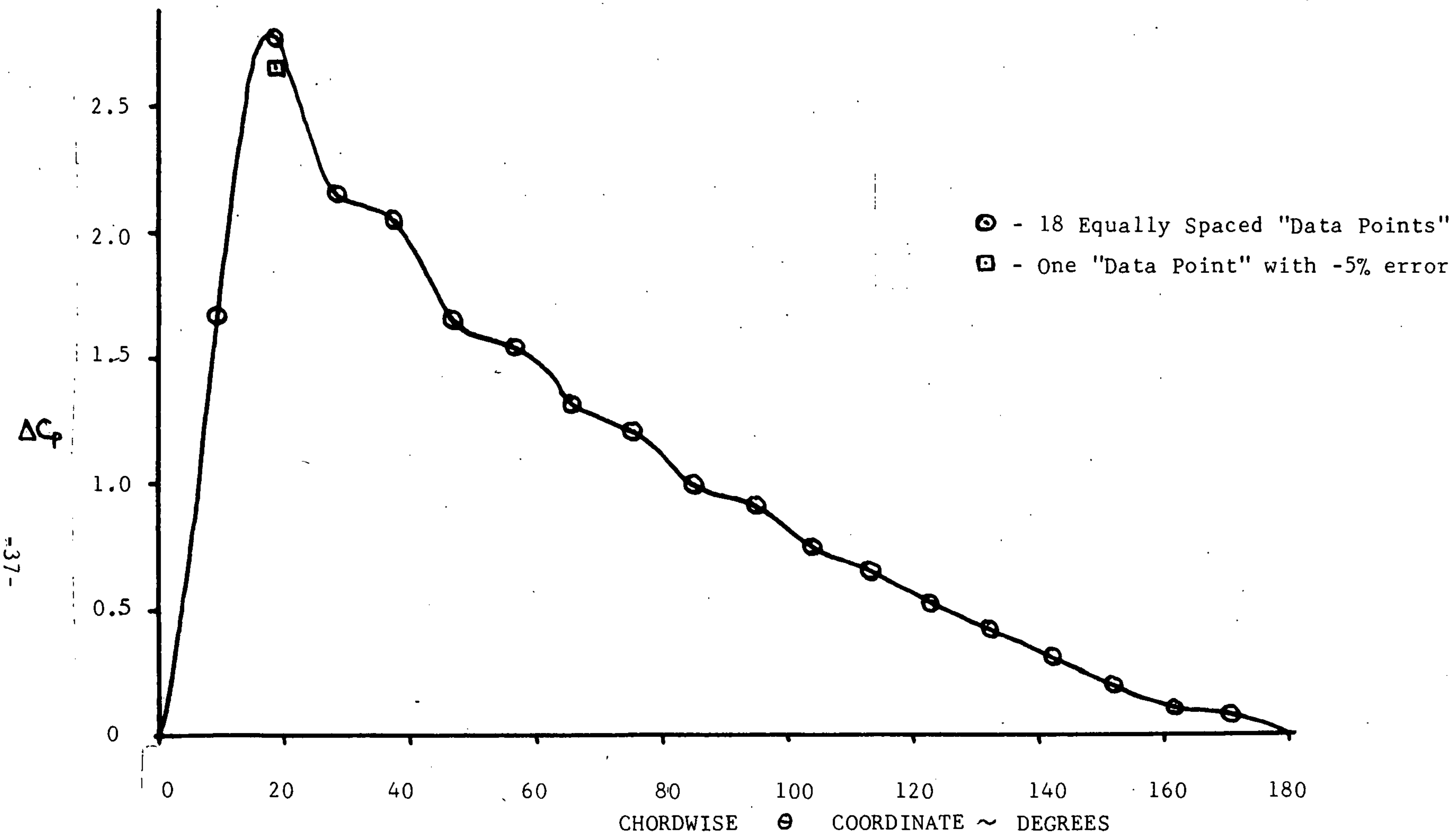

FIGURE 4. Example 2: 5% Error on One "Data" Point
Synthesized $\Delta C_p(\theta)$ of NACA 4415 Airfoil at $\alpha = 6°$

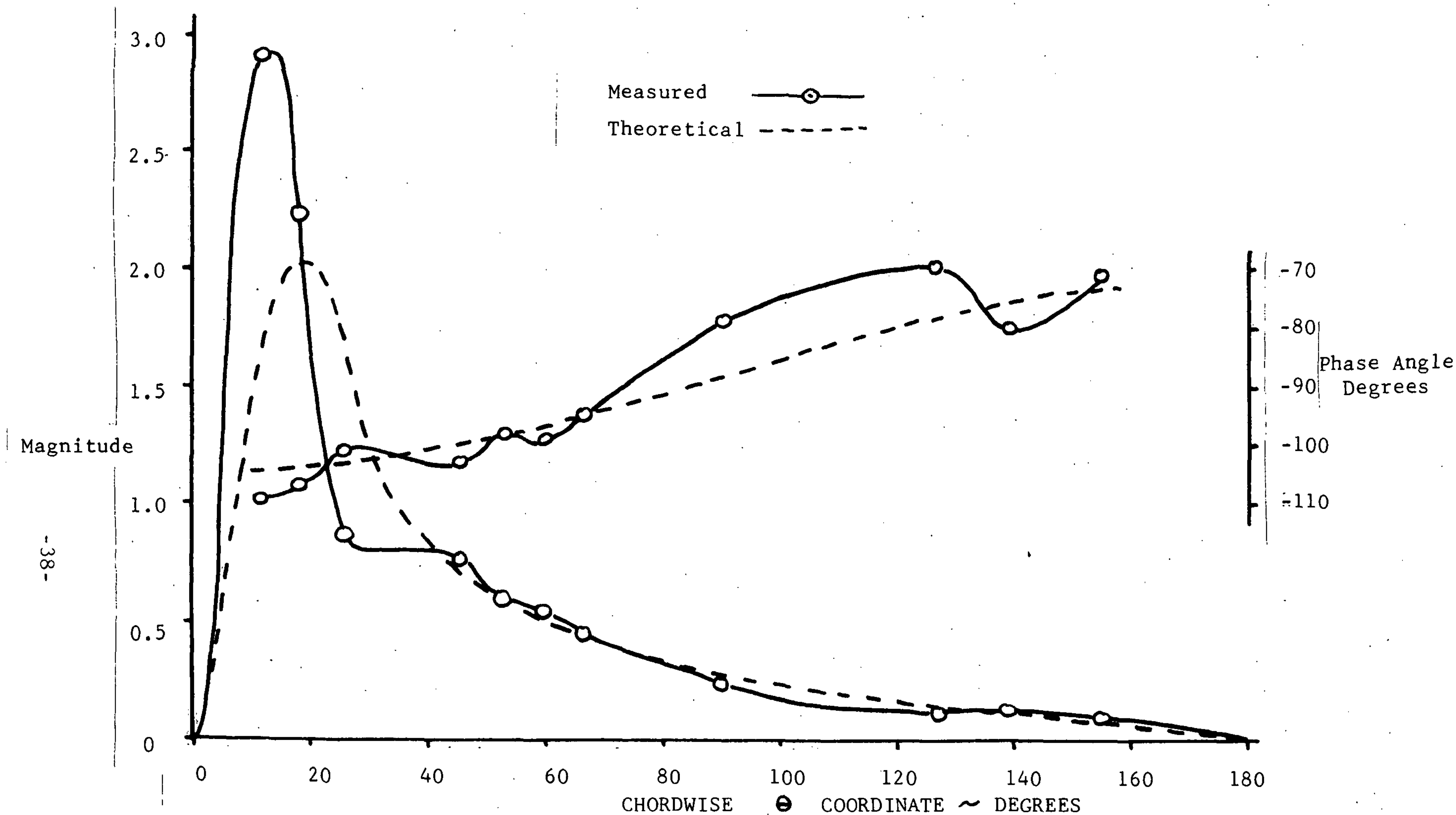

FIGURE 5. Theoretical and Measured Unsteady Pressure Distribution - Due to Oscillation in Plunge Only

APPENDIX I

EQUATIONS AND PROCEDURE FOR ANALYZING IN TERMS OF THE GLAUERT FUNCTIONS

The following is an outline of the equations and procedures for analyzing measured airfoil pressure distributions relative to the Glauert function-set.

The airfoil pressure distribution in terms of the Glauert functions (with the theoretical $\cot\theta/2$ angle-of-attack function replaced a semi-empirical $B(\theta)$ function) as obtained thin airfoil theory is

1) $$\frac{\Delta p(\theta,t)}{2\rho V} \equiv P(\theta,t) = A_0(t)\,B(\theta) + \sum_{m=1}^{M} A_m(t)\sin m\theta$$

$$+ \frac{b}{V}\frac{\partial}{\partial t}\Big\{\big[A_0(t) + \tfrac{1}{2}A_1(t)\big]\theta + \big[A_0(t) + \tfrac{1}{2}A_2(t)\big]\sin\theta$$
$$+ \frac{1}{2}\sum_{m=1}^{M}\frac{1}{m}\big[-A_{m-1}(t) + A_{m+1}(t)\big]\sin m\theta$$

If the operating conditions are periodic then the Glauert coefficients A_m for $m = 0,1,2,\cdots M$ can be represented as

2a) $$A_m(t) = a_0 + \sum_{n=1}^{N}\left(a_{mn}\cos n\omega t + b_{mn}\sin n\omega t\right)$$

thus

2b) $$\frac{\partial A_m(t)}{\partial t} = \omega\sum_{n=1}^{N} n\left(-a_{mn}\sin n\omega t + b_{mn}\cos n\omega t\right)$$

Now if the θ - term of (1) is expanded in a sine series, thusly

3) $$\theta = 2\sum_{m=1}^{M}\frac{(-1)^{m+1}}{m}\sin m\theta$$

and the result, (3), substituted in equation (1) along with equations (2), the result, at each harmonic, can be put in the following form:

4) $$P_n(\theta,t) = P_{nc}(\theta)\cos n\omega t + P_{ns}(\theta)\sin n\omega t$$

where

5) $$P_{nc}(\theta) = C_{on}B(\theta) + \sum_{m=1}^{M} C_{mn}\sin m\theta$$

$$P_{ns}(\theta) = S_{on}B(\theta) + \sum_{m=1}^{M} S_{mn}\sin m\theta$$

The coefficients C_{mn} & C_{ms} of (5) are then related to the components a_{mn} & b_{mn} of the Glauert coefficients $A_m(t)$ (equations (2)) by the following

6a) $$C_{on} = a_{on}$$

$$S_{on} = b_{on}$$

6b) $$C_{in} = a_{in} + k\left[3b_{on} + b_{in} + \tfrac{1}{2}b_{2n}\right]$$

$$S_{in} = b_{in} + k\left[3a_{on} + a_{in} + \tfrac{1}{2}a_{2n}\right]$$

6c) $$C_{mn} = a_{mn} + \frac{k}{m}\left[(-1)^{m+1}(2b_{on} + b_{in}) + \tfrac{1}{2}\left(-b_{(m-1)n} + b_{(m+1)n}\right)\right]$$

$$S_{mn} = b_{mn} + \frac{k}{m}\left[(-1)^{m+1}(2a_{on} + a_{in}) + \tfrac{1}{2}\left(-a_{(m-1)n} + a_{(m+1)n}\right)\right]$$

The procedure for analyzing the experimental $\Delta p(\theta,t)$ in terms of the Glauert function is:

1) Put the data in the dimensional form $P(\theta,t) \equiv \dfrac{\Delta p(\theta,t)}{2\rho V}$

2) Harmonically analyze the $P(\theta,t)$ to obtain the functional components $P_{nc}(\theta)$ and $P_{ns}(\theta)$ as per equation (4).

3) The $P_{nc}(\theta)$ and $P_{ns}(\theta)$ are analyzed as per equations (5) to obtain the coefficients C_{mn} & S_{mn}

4) The coefficients C_{mn} and S_{mn} are used in equations 6) to obtain the a_{mn} & b_{mn} components of the Glauert coefficients as in equation (4).

APPENDIX II

EQUATIONS AND ANALYSIS PROCEDURE FOR USING THE UNSTEADY PRESSURE RESPONSE FUNCTIONS (UPRF)

The following is an outline of the equations and procudure used for analyzing both steady and unsteady airfoil pressure distribution in terms of the UPRF.

Because the phase of the airfoil unsteady pressure response varies over the chord, the experimental $\Delta p(\theta,t)/2\rho V$ is resolved into two component functions, $P_{nc}(\theta)$ and $P_{ns}(\theta)$ by harmonically analyzing $\Delta p(\theta,t)$ at each chord location θ thusly;

1) $$\Delta p_n(\theta_i,t) = P_{cn}(\theta_i)\cos n\omega t + P_{sn}(\theta)\sin n\omega t\,;\quad i=1,2,\cdots NP$$

These two component pressure distributions $P_{nc}(\theta)$ & $P_{ns}(\theta)$ and define the chordwise distributions of the amplitude and phase of the unsteady pressure response of the airfoil relative to the observational reference system. If the airfoil stimulus (defined in section 3.1) at each harmonic is considered to be represented by the following cosine series

2) $$R_n(\theta,t) = \left(\sum_{j=0}^{M} r_j \cos j\theta\right)\cos(n\omega t+\gamma_j)$$

then it can be shown that the two pressure distribution components, $P_{nc}(\theta)$ and $P_{ns}(\theta)$ can be represented as follows

3a) $$P_{cn}(\theta) = N_{oc}F_0(\theta) - N_{os}G_0(\theta) + N_{1c}F_1(\theta) - N_{1s}G_1(\theta) + \sum_{m=1}^{M} C_{mc}\sin m\theta$$

3b) $$P_{sn}(\theta) = -N_{os}F_0(\theta) + N_{oc}G_0(\theta) + N_{1c}G_1(\theta) - N_{1s}F_1(\theta) + \sum_{m=1}^{M} C_{ms}\sin m\theta$$

where the $F_0(\theta)$, $G_0(\theta)$, $F_1(\theta)$, $G_1(\theta)$ and the Sine $m\theta$ are the components of the UPRF set (as defined in section 3.2.2). The expansions represented by equations (3) are the form of the analysis of the airfoil pressure distribution in terms of the UPRF. Each of the pressure distribution component functions $P_{nc}(\theta)$ and $P_{ns}(\theta)$ are analyzed (resolved) to obtain the coefficients of equations (3). The resulting coefficients of the analysis are related to the coefficients of the stimulus representation (equations 2) as follows

4a)

$$N_{oc} = r_o \cos \gamma_o$$

$$N_{os} = r_o \sin \gamma_o$$

$$N_{1c} = r_1 \cos \gamma_1$$

$$N_{1s} = r_1 \sin \gamma_1$$

4b)

$$C_{mc} = N_{mc} - \frac{k}{2m}\left(-N_{(m-1)s} + N_{(m+1)s}\right)$$

$$C_{ms} = N_{ms} - \frac{k}{2m}\left(-N_{(m-1)c} + N_{(m+1)c}\right)$$

where $N_{jc} = N_{js} \equiv 0$ for $j \leq 1$ and

4c)

$$N_{jc} = r_j \cos \gamma_j$$

$$N_{js} = r_j \sin \gamma_j \quad \text{for} \quad j = 2, 3, \cdots M$$

The procedure thus is:

1) The data must first put in the dimensional form

$$P(\theta,t) = \frac{\Delta p(\theta,t)}{2\rho V}$$

2) The $P(\theta,t)$ is harmonically analyzed to obtain the $P_{nc}(\theta)$ and $P_{ns}(\theta)$ as per equation (1).

3) The functions $P_{nc}(\theta)$ and $P_{ns}(\theta)$ are analyzed as per equations (3) to obtain the coefficents

4) The C_{mc} and C_{ms} are used in equations (4b) to obtain the N_{jc} & N_{js} for $j = 2, 3, \ldots M$

5) Finally, the N_{jc} and N_{js} for $j = 0, 1, 2, \ldots M$ are used in equations (4a) and (4c) to obtain amplitude, r_j , and phase , γ_j .